Parallel Kinematics

This document surveys parallel-kinematics literature and identifies its usefulness. The document has been developed while we were developing our SimParallel machine.

One of the aims of this document is to propose an effective solution to the limitations of the two rotary axes of five-axis machines that are currently used in industry. However, the survey of the parallel-kinematics literature will not be limited to this (two DOFs) family of parallel kinematics mechanisms lest a seed for an idea for our sought mechanism does exist in parallel-kinematics mechanisms with other DOFs. The available parallel mechanisms concepts will be mentioned and then their kinematics usefulness to our purpose will be more critically stated in the conclusion section.

The document consists of the following 11 sub-sections:

- Parallel-Kinematics Mechanisms.
- Six DOFs Parallel-Kinematics Mechanisms.
- Spatial Translational Three-DOFs Parallel-Kinematics Mechanisms.
- Spatial Rotational Three-DOFs Parallel-Kinematics Mechanisms.
- Other Three-DOFs Parallel-Kinematics Mechanisms.
- Asymmetric Parallel-Kinematics Mechanisms.
- Two DOFs Parallel-Kinematics Mechanisms.
- Four and Five-DOFs Parallel-Kinematics Mechanisms.
- Parallel-Kinematics Mechanisms Redundancy.
- Summary and Conclusions

1. Parallel-Kinematics Mechanisms

The conceptual design of PKMs can be dated back to the middle of the last century when Gough established the basic principles of a mechanism with a closed-loop kinematics structure and then built a platform for testing tyre wear and tear [Gough, 1956]. A sketch of the mechanism is shown in Figure 1. As shown in the figure, that mechanism allows changing the position and the orientation of a moving platform with respect to the fixed platform.
Later in 1965 Stewart designed another parallel-kinematics platform for use as an aircraft simulator [Stewart, 1965]. A sketch of that Stewart mechanism is shown in Figure 2. For some reason the mechanisms of Figure 1 and that of Figure 2 as well as many variations (e.g., the one shown in Figure 3) are frequently called in the literature Stewart platform. They are also called Hexapod mechanisms.
Of course other mechanisms related, may be less formally, to PKM existed well before and soon before Gough’s platform. Bonev [2003] surveyed many of these earlier mechanisms. Gough though is the one who gave some formalization to the concept. It might be interesting to know that Gough’s platform remained operational till year 1998 and it is now kept at the British National Museum of Science and Industry. See Figure 4 for photos of the original and current shapes of Gough platform.

Many have extensively analyzed Gough/Hexapod platform [Hunt, 1983, Fichter 1986, Griffis and Duffy, 1989; Wohlhart, 1994]. One problem with these six-DOF’s platforms is the difficulty of their forward-kinematics solution, because of the nonlinearity and the highly coupled nature of their governing equations. This difficulty has been overcome by introducing some assumptions [Zhang and Song, 1994] and a closed-form solution can be found in [Wen and Liang, 1994]. Others introduced some sensors to measure at least one of
the variables of the platform and hence reduce the unknowns of the governing equations [Merlet, 1993; Bonev et al, 1999]. The above mechanisms are six DOFs mechanisms because each of them allows the moving platform to move arbitrarily (within the limit of the workspace) in the six DOF space.

Having had a look on the mechanisms above one can now introduce a formal definition of parallel-kinematics mechanisms; A parallel-kinematics mechanism (or parallel manipulator) is a closed-loop mechanism. That is, a moving plate (ie end-effector) is connected to the stationary base by at least two independent kinematic chains, each of which is actuated. On the other hand, A serial-kinematics mechanism (or serial manipulator) is an open-loop mechanism in which each link is connected to ONLY two neighbouring links. All the mechanisms discussed in the introduction of Chapter 1 are serial-kinematics mechanisms.

The advantages of parallel-kinematics mechanisms in general are;

- Excellent load/weight ratios, as a number of kinematic chains are sharing the load.
- High stiffness, as the kinematics chains (limbs) are sharing loads and in many cases the links can be designed such that they are exposed to tensile and compressive loads only [Hunt, 1978]. This high stiffness insures that the deformations of the links will be minimal and this feature greatly contributes to the high positioning accuracy of the manipulator.
- Low inertia, because most of the actuators are attached to the base, and thus no heavy mass need to be moved.
- The position of the end-effector is not sensitive to the error on the articular sensors. Higher accuracy due to non-cumulative joint error.
- Many different designs of parallel manipulators are possible and the scientific literature on this topic is very rich, as will be shown later in this chapter.
- The mechanisms are of low cost since most of the components are standard.
- Usually, all actuators can be located on the fixed platform.
- Work-space is easily accessible.
- The possibility of using these mechanisms as a 6-component force-sensor. Indeed it can be shown that the measurement of the traction-compression stress in the links enables to calculate the forces and torques acting on the mobile platform. This is especially useful in haptic devices [Tsumaki et al, 1998].

On the other hand, the disadvantages of parallel-kinematics mechanisms in general are;

- For many configurations there are some analytical difficulties (eg the forward kinematics solution is not easy or finding all the mechanism singularities can be extremely difficult task).
- The need in many cases for the expensive spherical joints.
- Limited useful work-space compared to the mechanism size.
- Limited dexterity.
- Scaling up PKMs can enlarge the translational DOFs and usually is unable to enlarge the rotational DOFs.
- Potential mechanical-design difficulty.
- Mechanism assembly has to be done with care.
- Time-consuming calibration might be necessary. See [Ryu and Abdul-rauf, 2001] to realize that calibration of PKMs is not a trivial issue.
Many other different points of view about the benefits of PKMs and their drawbacks can be found in the literature [Brogårdh, 2002].

2. Six-DOFs PKMs

The PKMs mentioned in the previous section are six-DOFs PKMs. Some of these mechanisms have S-P-S kinematics chains. S-P-S chains are preferred as they, as discussed in Appendix A, transmit no torque through the limbs. These PKMs can also be realized using S-P-U chains or any other chain that has six-DOFs associated with its joints. One can check that against Grübler/Kutzbach criterion above, or review Appendix A. In fact a comprehensive attempt to enumerate the joints combinations and permutations that can be utilized when all the limbs are identical has been reported [Tsai, 1998]. It can also be shown that the DOFs associated with the limbs joints need to be at least six. See Appendix A too. Figure 5 shows one PKM that has been proposed. It uses six P-R-U-U limbs [Wiegand et al, 1996]. Similar to the PKMs above this one also has limited tilting ability. The reachable tilting angle changes strongly with the position of the P joints and fluctuates between 20 and 45 degrees. In special poses up to 57 degrees can be reached.

It is important to notice that changing the number of limbs in symmetrical six-DOFs PKMs will not change the DOFs of the platform. This has been shown using Grübler/Kutzbach criterion in Appendix A, and also can be observed in Figure 6 to Figure 9. In these examples though we need more than one actuator per limb, and if there are less than six actuators some of the DOFs will not be controllable.

A Symmetrical PKM is one that has identical kinematics chains (also called limbs or legs) each of which utilize identical actuator.

Figure 5
Six-DOFs PKM with six P-S-U limbs

Figure 6 shows another six-DOFs PKM [Tsai and Tahmasebi, 1993]. This PKM has three P-P-S-R limbs. However, planar motors can not provide high load-carrying capacity and they occupy the whole base leaving no space to the material to be processed. A similar PKM was later built and studied [Ben-Horin et al, 1996]. Figure 7 shows a PKM with three P-P-R-S limbs [Kim and Park, 1998; Ryu et al, 1998]. The range of tilting angles of the platform of this mechanism is one of the widest that can found in the literature. However, the mechanism uses 8 actuators (for the six P joints and two of the R joints) to realize the motion that can be realized with six actuators only, and many translational motions can be
realized in direct straight lines. A PKM similar to that shown in Figure 7 has been proposed earlier [Alizade and Tagiyev, 1994]. That earlier PKM had three P-R-P-S limbs instead.

Figure 6
Six-DOFs PKM with three P-P-S-R chains

Figure 7
Six-DOFs PKM with three P-P-R-S chains

Figure 8
Six-DOFs PKM using three Scott mechanisms
Probably no mechanism is more famous than the single DOF crank-slider of Figure 8.a. It is a P-R-R-R-R kinematic chain that converts linear to rotary motion or vice versa. Scott’s mechanism of Figure 8.b [Khurmi and Gupta, 1985] is another traditional planar mechanism that greatly resembles the well known crank-slider mechanism. Three of these Scott mechanisms have been put together, as shown in Figure 8.c, to realize a three-DOFs mechanism and then each of the Scott mechanism was made to displace vertically, resulting in a six-DOFs mechanism [Zabalza et al, 2002]. Some of the R joints of the original mechanism have been replaced by S joints to allow spatial motion of the arms. The advantage of the concept is that if one attempts to express the position and the orientation of the platform via its three vertices, then the kinematics relations will be fairly decoupled. The PKMs of Figure 6 and Figure 7 could be considered decoupled. Other six-DOFs decoupled PKMs have also been proposed [Zlatanov et al. 1992; Wohlhart, 1994].

Spherical actuators that can provide three-DOFs actuation are expensive and not commercially available [Williams and Poling, 2000], but if two of these actuators are used the Gough-Stewart platform of Figure 3 can be reduced to the one shown in Figure 9. Pending the quality and the mechanical characteristics of these spherical actuators, the solution offers an elegant and promising solution. The work-space, as least the translational part of it, is still limited though and load is shared between two rather than six limbs.

Six-DOFs PKMs represent the roots of the concept of PKMs and hence they had to be looked at. However, one might say that a six DOFs PKM is PKMs at their extreme, and consequently one might think that reducing the number of DOFs that act in parallel might alleviate the disadvantages of parallel-kinematics mechanisms while benefiting from their advantages. This is actually true in many cases. In trials to avoid the disadvantages of six DOFs parallel-kinematics mechanisms while utilizing the other benefits of parallel-kinematics mechanisms, two, three, four and five DOFs PKMs were proposed, as will be shown in the subsequent sections.

### 3. Spatial Translational Three-DOFs PKMs

Three-DOFs PKMs for pure rotation or pure translation are of special importance as they are, in our view, represent a low-level entity or building block of PKMs that helps deepening the understanding of these mechanisms. One can subsequently hybridize these two building
blocks or sub-systems from them. Spatial translational three-DOFs PKMs are discussed in this section and spatial rotational three-DOFs PKMs are discussed in the next section.

Using Grübler/Kutzbach criterion one can see that using three limbs (legs) each having five-DOFs is one way to realize three-DOFs symmetrical PKMs. See Appendix A for examples. Many of such PKMs have been built and Figure 10 to Figure 13 show examples of this family of translational three-DOFs PKMs. That is, Figure 10 shows a PKM with three limbs each with U-P-U joints [Tsai, 1996]. This mechanism has been studied by others [Di Gregorio and Parenti-Castelli, 1998] and been further optimized [Tsai and Joshi, 2000] and its mobility also has been discussed in details [Di Gregorio and Parenti-Castelli, 2002]. Obviously the same kind of motion can also be obtained using P-U-U kinematic chains [Tasora et al, 2001], as shown in Figure 11.
One should notice that the U-P-U mechanism is a special case from the R-R-P-R-R mechanism, when the axes of each R-R pairs are perpendicular. This R-R-P-R-R has been studied and the conditions that need to be satisfied to enable its pure translational motion has been established [Di Gregorio and Parenti-Castelli, 1998]. The P joints can also be replaced by R joints and the result is shown in Figure 12 [Tsai, 1999]. Alternatively one of the R joints could be replaced by P joints resulting in R-P-R-P-R (or R-C-C; C for Cylindrical) [Callegari and Marzitti, 2003]. This is shown in Figure 13.

In fact all the combinations and permutations the basic R and/or P joints that would result in PKMs with three five-DOFs limbs have been enumerated [Tsai, 1998; Kong and Gosselin, 2001]. Notice that if pure translation is sought using symmetrical PKMs, then S joints would not be a favorable choice as one S joint in each limb simply means that rotation cannot be constrained.

The Delta mechanism [Clavel, 1988] is one of the earliest and the most famous spatial translational three-DOFs parallel-kinematics robots, as it has been marketed and used industrially for pick and place applications. A sketch of this mechanism is shown in Figure 14. This mechanism can provide pure 3D translational motion to its moving platform using its three rotary actuators via its three limbs. Each of these limbs actually is a R-R-Pa-R
(Revolute-Revolute-Parallelogram-Revolute) kinematic chain. The mechanism can also provide a rotary independent motion about the Z axis as a 4th decoupled DOF.

Figure 14
Clavel-Delta translational PKM

Many variations of that Delta mechanism has been proposed and implemented. One of these close variations is the patented Tsai’s manipulator [Tsai, 1997; Stamper 1997], which also provides 3D translational motion to its platform. Here, the parallelograms are constructed using R joints instead of S joints and Stirrups in the previous case. That mechanism is shown in Figure 15. Another close variation was also presented [Mitova and Vatkitchev, 1991]. The kinematic chains of that variation were R-Pa-R-R instead.

Figure 15
Tsai or Meryland translational PKM

A P-R-Pa-R with vertical prismatic joints was also suggested [Zobel et al., 1996]. Variations extremely similar to that were later implemented using pneumatic drives [Kuhlbusch and Neumann, 2002]. These variations are shown in Figure 16.
When the lines of action of the three prismatic joints are tilted further till all of them are in the horizontal plane, the star mechanism of Figure 17 is then obtained. This mechanism was developed by Hervé [Hervé and Sparacino, 1992]. Notice here that the prismatic joints are replaced by helical ones (ie screw & nut), which should not represent a difference from kinematics points of view.
The orthoglide mechanism [Wenger and Chablat, 2000 and 2002] is another variation with the angles between the action lines of the prismatic joints are changed further resulting in better motion transformation (from joints to platform) quality. This is shown in Figure 18. Prior to that a similar mechanism has also been designed and built as a coordinate-measuring-machine [Hiraki et al, 1997]. In that mechanism the lines of action of the prismatic joints are changed further to guarantee that the heavy parts if the mechanism are resting on the machine base.

Parallelograms represent a common thread among the mechanisms of Figure 14 to Figure 18 as a parallelogram would directly constrain the rotational motion about certain axis. See Appendix A. Notice also that in all the designs above the two axes of the two revolute joints of each chain are always parallel, sometimes parallel to the direction of the prismatic joint (if any) and sometimes perpendicular to it, which agrees with conditions shown later in the literature [Kong and Gosselin, 2004b].

It is important to notice that each limb of each of the PKMs of Figure 14 to Figure 18 has only four-DOFs associated with its joints. According to Grübler/Kutzbach criterion these PKMs are not mobile [Stamper, 1997]. In fact some mechanisms are mobile only under some geometric conditions. These are called internally over-constrained mechanisms. See Appendix A for more about these over-constrained mechanisms. Screw theory can be utilized in conjunction with the Grübler/Kutzbach criterion [Huang and Li, 2002] to show the mobility of these over-constrained mechanisms.

Further, other (that do not utilize parallelograms) spatial translational PKMs with three limbs each of which having four-DOFs have been proposed. Symmetrical PKMs that have three (P-R-R-R) limbs and are aimed at realizing pure spatial translational motion have been built [Kong and Gosselin, 2002a; Kong and Gosselin, 2002b]. Two of these PKMs are shown in Figure 19. For these over-constrained PKMs to realize pure translation the following geometrical conditions need to be satisfied:

- The axes of the 3 R joints within the same limb are parallel.
- The three directions of the R joints of the limbs should not be in the same plane or parallel to the same plane.
- Within the same leg the axis of the P joint is not perpendicular to the direction of the R joints axes.
The directions of the P joints don’t have to be parallel, but if they are this will help enlarging
the work-space. Also, it has been shown that if the three directions of the R joints are
perpendicular to each other linear isotropic transformations will be obtained throughout the
work-space (and thus no singularities). Compare that to the isotropic conditions reported for
the orthoglide mechanism of Figure 18. Isotropic transformation is discussed further in
Chapter 4.

The geometrical conditions of the mobility of a similar over-constrained PKMs that has
three C-P-R (P-R-P-R) limbs, shown in Figure 20, have also been found [Callegari and
Tarantini, 2003]. These conditions are;

- The axes of the 2 R joints within the same limb are parallel.
- The three directions of the R joints of the three limbs should not be in the same
  plane or parallel to the same plane.

It has been shown that singularity of that mechanism can be kept outside the work-space
while maintaining a convex work-space. The isotropic points of that mechanism have also
been shown.

In fact the geometrical conditions of the different over-constrained PKMs that utilize four-
DOFs limbs have been enumerated [Hervé and Sparacino, 1991; Kong and Gosselin, 2004a].
Using three limbs each with P-P-P joints is actually another, may be trivial, over-constrained
translational PKMs.

Another concept that has been extensively utilized at the industrial level is presented now. If
three limbs each with six-DOFs (eg U-P-S kinematic chain) associated with its joints are
used, then the platform will have six DOFs (as discussed in Appendix A). However, if less
than six actuators are used with these three limbs then some DOFs will not be controllable.
After choosing which DOFs are to be controlled, one can compensate for the known but
uncontrolled motion of the remaining DOFs using other, may be serial, mechanism. One
can also use some limbs to mechanically constraint some of the platform DOFs. In fact this
is the basic idea behind Neumann’s patented mechanism [Neumann, 1988] of Figure 21.
This seems like creating some DOFs that are needed and then constraining or compensating
for them. Still, the idea has been utilized. Various aspects of this PKM has been studied
extensively [eg Siciliano, 1999] and a further utilization of the concept will be shown in a
subsequent section of this chapter. One might say or think that this concept/mechanism is
actually is under-utilization of resources because of a prior conviction to utilize a Gough-like
platform/limbs.
4. Spatial Rotational Three-DOFs PKMs

Exactly as in the case of spatial translational three-DOFs PKMs spatial rotational three-DOFs PKMs can be realized using three limbs each with five-DOFs associated with its joints. The difference now is how the joints of the PKM would be assembled. A PKM with three U-P-U limbs, just like the one discussed in conjunction with Figure 10, has been proposed [Karouia, and Hervé, 2000]. Another PKM with three R-R-S (or R-S-R) limbs, as shown in Figure 22, has also been proposed [Karouia, and Hervé, 2002a]. PKM with three R-U-U have also been presented as well [Hervé and Karouia, 2002b]. Figure 23 also shows how to use three P-R-P-R-R (or C-P-U) limbs to realize a spherical/rotation three-DOFs PKMs [Callegari et al 2004]. PKMs with three U-R-C and with three R-R-S legs have been proposed as well [Di Gregorio, 2001; Di Gregorio, 2002]. A PKM that utilizes parallelograms (similar to the delta PKM above) within its three R-Pa-S limbs was yet another proposed spherical PKM [Vischer and Clavel, 2000]. In fact the possible spherical PKMs that are based on five-DOFs limbs are enumerated [Kong and Gosselin 2004b; Kong and Gosselin 2004c; Karouia, and Hervé, 2002b; Karouia, and Hervé, 2003].
Again, as in the translational case, over-constrained PKMs can be used to realize orientational PKMs. If only R joints are used then three R-R-R legs can be used [Gosselin and Angeles, 1989]. The geometric condition that will mobilize this over-constrained PKM is that all the axes of the used R joints are to be concurrent at the rotation center of the mechanism. See Figure 24. Figure 25 shows one of these R-R-R limbs separately. Notice that in this case the space freedom (λ) is three as no element of the mechanism is translating, which should simplify the application of Grübler/Kutzbach criterion. Notice also that only two R-R-R legs can theoretically be used to realize a three-DOFs rotational PKM. See Appendix A. This is not usually favorable though as one actuator will not be placed on the PKM base. For isotropic transformation the axes of the R joints of each limb should be perpendicular to each other [Wiitala and Stanisić, 2000].
When P joints are used then four-DOFs legs can be used to realize over-constrained rotational PKMs [Kong and Gosselin, 2004c]. The combinations and permutations of possible over-constrained spherical PKMs as well as their necessary geometrical conditions are enumerated [Kong and Gosselin, 2004b; Kong and Gosselin, 2004c].

As happened in the translational case using Neumann’s PKM of Figure 21, three six-DOFs legs can be used to realize a six-DOFs PKM and then mechanically constrain the translational DOFs. The limbs used can have kinematic structure of P-U-S or R-U-S or their variations, as per Figure 26. In these cases an arm extending from the base is used to pivot/constrain the platform. The P-U-S or R-U-S chains can theoretically be replaced by S-P-S chain, which also has six DOFs associated with its joints [Mohammadi et al, 1993], as shown in Figure 27.

![Figure 26](orientation_u_p_s_or_r_u_s_pkm.png)

**Figure 26**
**Orientation U-P-S or R-U-S PKM**

![Figure 27](orientation_s_p_s_pkm.png)

**Figure 27**
**Orientation S-P-S PKM**
Type synthesis of three-DOFs rotational PKMs based on either Lie/Displacement group theory [Karouia and Hervé, 2003] or on screw theory [Kong and Gosselin, 2004b] have been discussed.

5. Other Three-DOFs PKMs

So far spatial three DOFs mechanisms have been discussed. Three DOFs mechanisms can provide planar motion too. That is, they can provide the platform with two translational motions and one rotational motion about the plane normal. If, one relies on P and/or R joints as well as Grübler/Kutzbach criterion, then one can find that there are 7 possible symmetrical mechanisms. These are (RRR, RRP, RPR, PRR, RPP, PRP, and PPR). S and U joints here not useful here. Each of the three identical kinematic chains in this case needs to have 3 DOFs [Tsai, 1998]. Figure 28 [Hunt, 1983] and Figure 29 [Tsai, 1998] represent two of these possible seven mechanisms that have actually been implemented.

The mechanism of Figure 30 is another planar symmetrical 3 DOF PKM that has been proposed [Marquet et al, 2001]. Three P-R-R limbs are used. In the figure one can actually see a 4th chain. This is actually a redundant one to treat singularity, which will be discussed in Section 7. With this fourth P-R-R limb P-U-S limbs have also been proposed.

Planar PKMs cannot provide two spatial rotational DOFs and hence they can not directly serve the purpose of this work, and hence they are surveyed thoroughly. Other PKMs can
provide one translational and two spatial rotational DOFs. These PKMs should be combinations of the pure rotation and pure translation described above. Example is the R-P-S mechanism [Hunt, 1983; Lee and Shah, 1988a; Lee and Shah, 1988b] that is shown in Figure 31. According to Grübler/Kutzbach criterion this mechanism provides three-DOFs. This mechanism can be assembled to control one, two or three rotational DOFs of the platform. The remaining controlled ones would be translational DOFs.

**Figure 31**

R-P-S PKM for two rotations and one translation

To realize one translational and two rotational DOFs a P-R-S PKMs was also proposed [Merlet, 1991]. This is shown in Figure 32. Replacing the prismatic joints by revolute ones would maintain the mechanism's three-DOFs. This R-S-R PKMs is shown in Figure 33 and has actually been implemented [Hui, 1995; Dunlop and Jones, 1997]. Also, a small variation of that mechanism was earlier patented [Lambert, 1987]. Again, the revolute joints have been arranged to control one translational and two rotational DOFs.

**Figure 32**

P-R-S PKM for two rotations and one translation

**Figure 33**

R-S-R PKM for two rotations and one translation
In fact all the possible combinations and permutations that can result in five-DOFs limbs can theoretically be used to obtain one translational and two rotational DOFs after proper joints arrangements. Not all these cases have been reported in the literature though. Obviously, some of these variations might not be favourable from manufacturing point of view, and some arrangements or joints might limit the work-space.

One other useful idea in PKM is separating actuation from constraining. For example, the U-P-U PKM of Figure 10 will not be able to move except in the three translational directions because of the way its joints are arranged. One can now use other limbs for actuation. For example, in Figure 34 the three P actuators of three S-P-S limbs are used for that purpose. Notice, however, that these actuating limbs should not impose any constraint on the motion of the platform. As actuation can be separated from motion constraining also rotational motion can be separated from translational motion [Tsai, 1998].

6. Asymmetrical PKMs

All of the above mechanisms are Symmetrical PKMs. That is, each of these mechanisms has a number of identical kinematic chains (limbs) each with a similar actuator. Obviously non-symmetrical DOFs PKMs can also be developed, and the variations will then be infinite. Among the asymmetrical three-DOFs PKMs that have actually been reported are ones that can realize three translational spatial DOFs, one rotational and two translational DOFs [Liu et al., 2002] and three spatial rotational DOFs [Cheng, 1994; Agrawal et al, 1995]. As an example, these spatial rotational three DOFs are shown in Figure 35.

Symmetry in PKMs represents one of its powerful traits. This is because of the resulting modularity and hence simplicity and cost saving. Also, a symmetrical mechanism is likely to have equal properties within its operating range. More importantly, no specific gain was found in the asymmetrical literature that makes us think about sacrificing the gains obtained from symmetrical PKMs.
7. Two DOFs PKMs

The famous planar five-bar mechanisms of Figure 36 or its variations that utilize P joints instead of some of the R joints (see Appendix A) represent the most commonly used 2 DOFs PKMs. This is a planar mechanism that can provide two planar rotational (or translational) DOFs [Liu et al., 2002; Majou et al., 2002].

A spatial five-bar mechanism would result in only one DOF such as Bennett’s mechanism [Gracia, 1999]. See Appendix A. Figure 37a [Hervé, 2004] shows how to realize two spatial rotations (rather than the planar ones of Figure 36). Notice that for this over-constrained mechanism only one actuator can be placed on the machine base. Figure 37b shows another over-constrained version that would allow placing both actuators on the machine base [Gosselin and Caron, 1999].

Figure 38 shows the situation when P joints are used [Carricato and Parenti-Castelli, 2004]. The mechanism is asymmetrical just like the one of Figure 37b. One limb is P-R-R-R and the other is P-R-R-R-R. One should notice that in all cases the axes of rotations of both limbs have a common center/point.
Theoretically, according to Grübler/Kutzbach criterion, one can not realize a non-over-constrained spatial symmetrical PKM with two-DOFs. Instead one can utilize Neumann’s idea (of Figure 21.) above [Carricato and Parenti-Castelli, 2004]. This is shown in Figure 39.

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8. Four and Five-DOFs and PKMs

In this work we are not interested in four and five-DOFs PKMs per se. However, we are interested in spatial rotational two-DOFs PKMs or PKMs that combine these two spatial rotational DOFs with one or more translational DOFs, which can be studied through studying the literature of these four and five-DOFs PKMs.

Theoretically, in general, it is not possible to realize symmetrical non over-constrained four or five DOFs PKMs. In more details; many of the spatial three-DOFs PKMs above have three limbs each of which has five-DOFs associated with its joints (eg S-P-R, R-U-U …etc). If one simply adds extra identical limb to one of these symmetrical three-DOFs mechanisms (say the one of Figure 11) the result will not be a four-DOFs PKM. This has been shown in Appendix A using Grübler/Kutzbach criterion. Also, it has been shown in the same appendix that using 4 identical limbs each with six-DOFs associated with its joints (eg, U-P-S similar to the ones possible used in Gough-Stewart Platform) will not result in a four-DOFs platform. Exactly the same argument applies to five-DOFs PKMs. Therefore, it is not straight forward to realize symmetrical four or five-DOFs PKMs. This is probably the reason behind the scarcity of four and five DOFs PKMs in the literature.

It is possible, on the other hand, to extend Neumann’s idea, of Figure 21, to four or five-DOFs PKMs. That is, use a mechanism similar to Gough’s or actually to that of Figure 3 but with only four/five limbs and then mechanically constraint the two remaining uncontrolled DOFs. Exactly the same has been proposed [Zamanov and Satirov, 1992] for five-DOFs PKMs. The obvious problem would be the limited operating range.

It has also been proposed to use four identical limbs each of which is a P-U-U chain, connect each pair of these chains to an intermediate rigid body, and then connect these two intermediate rigid bodies to the platform using two R joints [Company and Pierrot, 1999]. These kinematic chains look as shown in Figure 40. Noticing that a U-U chain can be replaced by a spherical joints and bars exactly as has been done in the Delta mechanism above, then one version of the actual mechanism proposed would like as shown is Figure 41. When R joints are used instead, then the resulting mechanism is shown Figure 42.

Others [Pierrot et al, 2001] suggested to replace the above kinematics chains by the ones shown in Figure 43. That is, two of the chains have two S joints instead of the original U joints. The actual resulting mechanism is shown in Figure 44, which is the four-DOFs version of the hexa-glide of Figure 5. The version that utilizes R joints instead of the P Joints is shown in Figure 45. These 4 PKMs have been designed to provide one rotational and three translational DOFs. The mechanisms suffer from limited range of the rotational DOF (90 degrees). Subsequently it has been suggested [Pierrot and Company, 2000] to use gears to amplify the rotational operating range. The 5th rotational DOF is to be realized in the conventional way discussed in Chapter 1 of our current work.
Figure 40
Kinematics Chains of one
Four-DOFs PKM

Figure 41
P joints Four-DOFs PKM

Figure 42
R joints Four-DOFs PKM

Figure 43
Kinematics Chains of another
Four-DOFs PKM

Figure 44
Four-DOFs PKM with
P-U-U, P-U-U, P-U-S and P-U-S
Chains
Another asymmetrical four-DOFs PKMs has been proposed [Clavel et al, 2002] and is shown in Figure 46. This work started by listing useful criteria that need to be adopted to achieve industrially attractive five DOFs machine. The work ended though with a four-DOFs PKMs, one rotational and three translational DOFs. Again, a conventional rotational DOF has been suggested as a fifth DOF. Figure 47 shows the joints connections of the seven kinematics chains used. The additional chains have been used to widen the operating range of the rotational DOF of the mechanism till it became 120 degree. Replacing the P joint of this mechanism by R joints is, as usual, possible and the result is shown in Figure 48.
It might be obvious that one can augment any of the pure rotational or pure translational PKMs discussed above with one or two DOFs. An example of that is shown in Figure 49 where the three rotational DOFs of Figure 24 are augmented with a translational DOF [Zlatanov and Gosselin, 2001]. On advantage of this approach is that the kinematics of rotational and translational DOFs are likely to be decoupled, which is likely to simplify the mechanism analysis.

On the other hand, symmetrical over-constrained PKMs with four or five DOFs can be built. However, this is not usually an optimal choice. Figure 50 shows a PKM that can provide three rotational DOFs (notice the concurrent axes of the R joints) and one translational DOF [Zlatanov and Gosselin, 2001]. In fact when this PKM was optimized for better work-space the PKM of Figure 49 was the result. Three limbs and the middle mast are used, as in Figure 21, to constrain the uncontrolled DOFs. Figure 51 shows another four (this time one rotational and three translational) DOFs PKMs.
Figure 52 shows five (three rotational and two translational) DOFs PKM. Notice the concurrent R joints that are to provide the three rotational DOFs. Figure 53 shows another five (this time two rotational and three translational) DOFs PKM. This time P joints are used. Notice that in every leg each two R joints have parallel axes, just like over-constrained three or four DOFs translational DOFs.

In fact the possible (i.e., the above and other) over-constrained symmetrical four and five-DOFs PKMs have been enumerated, and the geometrical conditions that will mobilize these PKMs also been discussed [Fang and Tsai, 2002].
9. PKMs Redundancy

Adding one or more actuators in addition to the ones needed to realize certain DOFs is called actuation redundancy. This principle has been historically used to realize extra objective, e.g., compensate for backlash [Kwon et al., 2000], vibration or resonances (i.e., extreme loss of stiffness) control [Chawla et al., 1994], obstacle avoidance or manipulability improvement [Stadler, 1995].

Actuation redundancy is also used in many PKMs. In some cases, it is done merely to share load among more actuators. The principle is more meaningful though when used to improve manipulability or in extreme cases to treat the lack of stiffness (gain of DOFs) that happens at the singular positions. Manipulability and Singularities are discussed in more details in Chapter 4.

In fact Figure 7 and Figure 30 show two PKMs that utilize redundancy to overcome or to shift singularity. The PKM of Figure 30 has four actuators to realize planar three (two translational and one rotational) DOFs, and the PKM of Figure 7 has extra two actuators to cancel two singularities. A redundant PKM similar to the Delta mechanism was also proposed [Reboulet et al., 1992]. This mechanism was realized industrially.

The previous was actuation redundancy. Kinematics redundancy (i.e., adding links and/or joints to the mechanism more than what it actually requires for its mobility) has also been utilized to achieve extra objective(s). In PKMs that extra objective has been reducing the mechanism singularity(s). It has been shown how to add one R joint to one leg of a PKM similar to the one shown in Figure 26.a to reduce the number of singularities of that PKM [Wang and Gosselin, 2004]. The result is an asymmetrical PKM though. The approach is neither systemized nor generic yet.

10. PKMs In Industrial Machine Tools

PKMs have been used in industrial applications and this is gaining momentum every day. Figure 54 to Figure 58 show some early and direct uses of Gough-Stewart platform as a milling machine.
Figure 54
Variax
By Giddings&Lewis, USA
www.glmachinetools.com

Figure 55
CMW 300
By Rozières-sur-Mouzon, France
www.hexapode.com

Figure 56
VOH-1000
By Ingersoll, USA
http://www.ingersoll.com/ind/hexapod.htm
It is obvious how limited is the work-space compared to the overall machine dimensions. Also, the range of the two tilting angle are limited. Even some of these machines can only provide four-sided five-axis contouring. One would also have difficulty stating that the mechanism is simple and easy to construct, a matter that motivated PKM in the first place.

HexaM of Figure 59 is a five-DOFs milling machine. Each of the six limbs is a P-U-S kinematic chain. It can only provide ± 30 degrees tilting of the head. Figure 60 shows the industrial version of the hexaglide that has been discussed earlier. See Figure 5.
The basic concept behind eclipse has been discussed above (Figure 7) and the industrial version of the machine is shown in Figure 61.
The Hexa Robot of Figure 62 has been invented at LIRMM as a kind of 6-axis version of the Delta. First prototypes have been built by the TOYODA Company in 1991 in cooperation with Tohoku University and Lirmm. Each of the limbs is an R-S-S kinematics chain.

Figure 63 shows one of the many commercial variations of Delta robot of . It provides translational three-DOFs.
The translational P-R-Pa-R of Figure 16 has been used to provide three translational motions to a turning machine. One can also place a rotating table instead of the turning spindle. This is shown in Figure 64.

Figure 65 shows another variation of the previous machine where all the P joints have the same line of action. This results in translational three-DOF's with accessible work space.
Figure 65
Pegasus
By Reichenbacher, Germany
www.reichenbacher.de/site.asp?breite=1024

Figure 66 shows the industrial implementation of the P-U-S system of Figure 32. This is used to realize one translational and two rotational DOFs.

Figure 66
Ecospeed
By DS Technologie & Cincinnati
Figure 67 shows another similar implementation to the same previous concept. It uses 6 limbs though. Only three DOFs are actuated and hence controlled. Ulsys of Figure 68 utilizes a concept that is similar to Neumann’s of Figure 21. P-U-S limbs are used and hence the PKM has six DOFs. Three of these are constrained using a universal joint and a sliding mechanism.

The Tricept of Figure 69 and the DMT of Figure 70, again, utilize a concept that is similar to Neumann’s of Figure 21. U-P-S limbs are used and hence the PKM has six DOFs. Instead
of constraining the three non-actuated DOFs only one is constrained using the middle rod and the other two are compensated for using a conventional 2-axis rotational head.

Figure 69
Tricept

By Neos, Sweden
www.neosrobotics.com

Figure 70
DMT 100
By DECKEL MAHO, Germany
www.fps-service.de
The scissor-like part of this machine is the simple five-bar mechanism. It is planar (not a spatial) mechanism, but is still considered PKM and is used to provide two translational DOFs.

Needless to say; there are many other industrial implementations of PKMs. The above are the industrial applications that thought to be not repetitive and present some point to our argument.

11. Summary and Conclusions

There are literally hundreds of PKMs and tens of PKMs patents in the literature. One can study these PKMs from many points of view and here the focus was on how to utilize this literature to help realizing a design of the two rotational DOFs that is superior to the existing ones. Many of the available PKMs are very similar, and the above was the author’s attempt to capture all the basic relevant kinematic concepts from the point of view just mentioned. The mechanisms reviewed should be looked at again after the analysis of Chapter 4 and Appendix A.

Six-DOFs PKMs represent the origin of the parallel-kinematics concept, but they have many limitations that have been outlined above. Six-DOFs PKMs are PKMs in the extreme and hence they bring all disadvantages of PKMs. Looking at three-DOFs PKMs that can provide spatial pure rotation or spatial pure translation would provide further deep insight into the parallel-kinematics concept, and would also allow understanding what the concept can practically offer. The over-constrained mechanisms that correspond to these three-DOFs pure translation or pure rotation PKMs are also useful to consider as they show the minimum amount of links and joints that can be used to realize the targeted motion.

If one is attempting to exploit the advantages and avoid the disadvantages of PKMs one might adopt the following recommendations;

- Place the actuators on the machine base to reduce the amount of moving masses. This means that only PKMs with number of limbs that equal the number of DOFs are to be considered.
- Use identical limbs to promote modularity, reduce cost, simplify construction and simplify kinematics analysis. Hence only symmetrical or very close to symmetrical PKMs are considered.

Try to minimize the number of limbs if decent rotational work-space is sought. The PKMs surveyed show that using more limbs will only add mechanical rotational constraints as well as possible links’ interferences and hence reduce the rotational work-space. The PKM of Figure 7 provides one of the largest rotational DOFs work-space in the literature. Although the mechanism is a six-DOFs one only three limbs were used.

Avoid PKMs that have more than one translational DOF if large translational work-space is sought, as this will invariably result in limited translational work-space in at least some of the resulting translational DOFs or lack of mechanism stiffness. This is evident from the various laboratory-scale and industrial PKMs discussed above. If one attempts to realize two or three translational DOFs using a PKM the links and the limbs will be impractically (from the point of view the weight that is needed to maintain stiffness) long (as per Figure 5 and Figure 10 to Figure 16) or the work-space will be small compared to the machine dimensions (as per Figure 17). An exception to the wording of this recommendation might be the cases of Figure 6 and Figure 7. However, these two PKMs also have the disadvantages discussed above. Also, the PKM of Figure 7 does utilize two actuators/limb and hence it violates the 1st recommendation above.

- Use traditional P joints that are not realized using extensible limbs to realize translational DOFs with decent work-space. These P joints have never represented a problem in machine construction, as linear bearings/guideways are standard off-the-shelf items that are available for any load and reasonable prices.

These recommendations mean eliminating many of the theoretically possible PKMs combinations and permutations. Further eliminations can be based on the facts that; six-DOFs PKMs provide more freedom than is sought in this work and four and five-DOFs PKMs, unless they are over-constrained, can not be symmetrical. These four and five-DOFs PKMs have been surveyed to get insight into the ways rotational and translational DOFs can be combined.

The actual aim of surveying PKMs in this work is to be able to replace the problematic two serial rotational DOFs that are currently and have traditionally been used in five-axis machines. This means that a two-DOFs PKM might be the solution to this situation. However, according to the recommendations above, utilizing a translational DOF in conjunction with these two rotational DOFs seems to add the benefit of distributing the load among three rather than two limbs without introducing any drawback. The result would be a three-DOFs PKM with one translational and two rotational DOFs.

It seems that all the above leads to the conclusion that utilizing three-DOFs PKM that is hybridized with two conventional cartesian/serial DOFs represent the best of all worlds in the five-axis machine universe. This somehow is supported by intuition too as it is not plausible that a relatively new or less formalized idea such as PKM would come and totally replace serial kinematics that have been used, usually successfully, for long time.
To realize a three-DOFs symmetric PKM according to the above recommendations one can use three limbs each with five or four DOFs. Using six-DOFs limbs is also possible but is not as easy. Using four-DOFs limbs will result an over-constrained mechanism that has not been studied before. Over-constrained mechanisms are stiffer and simpler to construct mechanisms, as they have fewer joints and links. A corresponding non-over-constrained mechanism would have an extra R (or P) joint per limb. The price paid for this simplicity advantage is the requirement to perform more accurate force-analysis and to follow more strict manufacturing tolerances. This is because links and joints of over-constrained mechanisms (as would intuitively be expected) are subjected to higher loads, as some links are to resist loads that are actually passed by the missing joints.

The literature contains over-constrained PKMs for pure translation or pure rotation. Combined rotation-translation three-DOFs using over-constrained PKMs have not been discussed before in the literature. The main problem with the non-over-constrained rotational-translational three-DOFs PKMs (such as the ones shown in Figure 31, Figure 32, Figure 33 and Figure 66) is their heavily coupled kinematics as well as the displacement associated with the rotations which renders the rotational motion actually helical motion. The singularities limit their work-space too.

Figure 37, Figure 38 and Figure 39 will allow realizing two rotational DOFs but each of them will have a problem if one attempts to incorporate additional translational DOF(s). Also, the one of Figure 39 has obvious space utilization problem.

The following are some other fairly unrelated observations on the PKMs literature;

- Actuated S joints are rarely used (see Figure 9) so are actuated U joints as they are not commercially available. These actuators can change the way PKMs are constructed.
- In using four or five-DOFs limbs to realize three-DOFs PKMs the way the joints are geometrically arranges will determine which three of the available six DOFs is to be obtained.
- Lie-Group [Hervé, 1992; Hervé, 1999] and screw theories provide important insight into the synthesis of PKMs. These theories to some extent systemize the way over-constrained PKMs can be designed.

The above conclusions represent the basis and the justification of the novel kinematic design presented in Chapters three and four.